Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. A basketball player makes 70% of her free throws. She takes 7 free throws in a game. If the shots are independent of each other, the probability that she makes 5 out of the 7 shots is about
   (a) 0.635.
   (b) 0.318.
   (c) 0.015.
   (d) 0.329.
   (e) 0.245.

2. It has been estimated that as many as 70% of the fish caught in certain areas of the Great Lakes have liver cancer due to the pollutants present. A sample of 130 fish is caught and inspected for signs of liver cancer. The number of infected fish within two standard deviations of the mean is
   (a) (81, 101).
   (b) (86, 97).
   (c) (63, 119).
   (d) (36, 146).
   (e) (75, 107).

3. In a triangle test a tester is presented with three food samples, two of which are alike, and is asked to pick out the odd one by tasting. If a tester has no well-developed sense of taste and can pick the odd one only by chance, what is the probability that in five trials he will make four or more correct decisions?
   (a) 0.045
   (b) 0.004
   (c) 0.041
   (d) 0.959
   (e) 0.955

4. A set of 10 cards consists of 5 red cards and 5 black cards. The cards are shuffled thoroughly and you turn cards over, one at a time, beginning with the top card. Let X be the number of cards you turn over until you observe the first red card. The random variable X has which of the following probability distributions?
   (a) The Normal distribution with mean 5
   (b) The binomial distribution with \( p = 0.5 \)
   (c) The geometric distribution with probability of success 0.5
   (d) The uniform distribution that takes value 1 on the interval from 0 to 1
   (e) None of the above

5. Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?
   (a) 0.965
   (b) 0.035
   (c) 0.989
   (d) 0.011
   (e) 0.736

6. Refer to the previous problem. Planners need to have enough beds available to handle a proportion of all outbreaks. Suppose a typical outbreak has 100 people exposed, each with a 40% chance of coming down with the disease. Which is not correct?
   (a) This scenario satisfies the assumptions of a binomial distribution.
   (b) About 95% of the time, between 30 and 50 people will contract the disease.
   (c) Almost all of the time, between 25 and 55 people will contract the disease.
   (d) On average, about 40 people will contract the disease.
   (e) Almost all of the time, less than 40 people will be infected.
7. There are 10 patients on the neonatal ward of a local hospital who are monitored by 2 staff members. If the probability of a patient requiring emergency attention by a staff member is 0.3, what is the probability that there will not be sufficient staff to attend all emergencies? Assume that emergencies occur independently.

(a) 0.3828
(b) 0.3000
(c) 0.0900
(d) 0.9100
(e) 0.6172

8. In 1989 Newsweek reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming that a class in a school is a random sample from the population of all children at risk, the probability that more than 3 children have to be tested until one is found to have a blood level that may impair development is

(a) 0.064.
(b) 0.096.
(c) 0.64.
(d) 0.16.
(e) 0.88.

9. Which of the following are true statements?

I. The histogram of a binomial distribution with \( p = 0.5 \) is symmetric. √
II. The histogram of a binomial distribution with \( p = 0.9 \) is skewed to the right. not always
III. The histogram of a geometric distribution with \( p = 0.4 \) is decreasing.

(a) I and II
(b) I and III
(c) II and III
(d) I, II, and III
(e) None of the above gives the complete set of complete responses.

10. Binomial and geometric probability situations share many conditions. Identify the choice that is not shared.

(a) The probability of success on each trial is the same.
(b) There are only two outcomes on each trial.
(c) The focus of the problem is the number of successes in a given number of trials.
(d) The probability of a success equals 1 minus the probability of a failure.
(e) The mean depends on the probability of a success.
Part II – Free Response (Question 11-13) – Show your work and explain your results clearly.

11. Brady, Ms. Wood’s favorite dog, loves to play catch. Unfortunately, Brady is not particularly adept at catching as his probability of catching the ball is 0.15.

(a) Ms. Wood is interested in determining how many tosses it will take for Brady to catch the ball once.

(i) Can this situation be described as binomial, geometric, or neither?

\[ \text{geometric } p = 0.15 \]

(ii) What is the mean & standard deviation of the number of catches?

\[ \mu = \frac{1}{0.15} = 6.67 \text{ tosses} \]

\[ \sigma = \sqrt{\frac{0.85}{0.15^2}} = 6.15 \text{ tosses} \]

(iii) What is the probability it will take 10 tosses in order for Brady to catch the ball?

\[ P(X = 10) = (0.85)^9 (0.15) = 0.035 \]

(b) Mr. Greenberg, avid baseball player & coach, decides to train Brady. After three-a-day training sessions for 4 weeks, the probability that Brady catches the ball has increased to 0.35. Mr. Greenberg is interested in determining the number of times Brady will catch the ball in 25 tosses.

(i) Can this situation be described as binomial, geometric, or neither?

\[ \text{binomial } n = 25 \quad p = 0.35 \]

(ii) What is the mean and standard deviation of the number of catches?

\[ \mu = (25)(0.35) = 8.75 \text{ catches} \]

\[ \sigma = \sqrt{(25)(0.35)(0.65)} = 2.385 \]

(iii) What is the probability that Brady will catch the ball 8 times in 25 tosses?

\[ P(X = 8) = \binom{25}{8} (0.35)^8 (0.65)^{17} = 0.1007 \]

(c) Mr. Myers, knowing that Brady is a “learning” dog, determines that the probability that Brady will catch the ball on the first throw is 0.50 (After all, either he catches it or he doesn’t!!), but his probability of catching the ball improves by 0.05 on each subsequent toss. Mr. Myers would like to find out the number of tosses required for Brady to catch the ball three times.

(i) Can this situation be described as binomial, geometric, or neither?

\[ \text{neither} \]

(ii) What is the probability that four tosses are required for Sophie to catch the ball three times?

\[ P(X = 4) = 0.0715 + 0.0878 + 0.1073 = 0.2665 \]

\[ \begin{array}{c}
\checkmark \checkmark \checkmark \\
(0.50)(0.55)(0.40)(0.65) = 0.0715 \\
\checkmark \checkmark \checkmark \\
(0.50)(0.55)(0.60)(0.65) = 0.0878 \\
\checkmark \checkmark \checkmark \\
(0.50)(0.55)(0.60)(0.65) = 0.1073
\end{array} \]
12. A quarterback completes 40% of his passes.

(a) Explain how you could use a table of random digits to simulate this quarterback attempting 10 passes.

Check sets of 10 digits

(b) Using your simulation scheme, perform your simulation 4 times. Using the random digit table below, begin on line 149 and circle the “successes.” For each simulation, calculate the percent of passes completed.

<table>
<thead>
<tr>
<th>Simulation 1 = 50%</th>
<th>Simulation 3 = 40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 149</td>
<td>71546 05233 53946 68743 72460 27601 45403 88692</td>
</tr>
<tr>
<td>Simulation 2 = 40%</td>
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</tr>
<tr>
<td>Line 150</td>
<td>07511 88915 41267 16853 84569 79367 32337 03316</td>
</tr>
</tbody>
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13. The number of sarcastic comments Mr. Myers makes during a typical school day is a random variable that is approximately normally distributed with a mean of 15 and a standard deviation of 5. The number of times Ms. Wood laughs uncontrollably during a typical school day is a random variable that is determined by flipping a coin 10 times and determining how many heads occur.

(a) An “impressive” day is defined for Mr. Myers by making at least 20 sarcastic comments. An “impressive” day is defined for Ms. Wood by laughing uncontrollably at least 8 times. Find the probability of an “impressive” day for each teacher.

Mr. Myers

\[ P(X \geq 20) = .1587 \]

Ms. Wood

\[ P(X \geq 8) = 1 - P(X \leq 7) = .0547 \]

(b) Over the next five days, determine the probability of 3 “impressive” days for Mr. Myers and Ms. Wood having her first “impressive” day on the fifth day.

Mr. Myers

\[ P(X = 3) = \binom{5}{3} \left(\frac{5}{3}\right)^3 \left(\frac{1}{3}\right)^2 = .0283 \]

Ms. Wood

\[ P(X = 5) = (.0547)(.9453)^4 = .0437 \]

\[ P(M \text{ and } W) = (.0283)(.0437) = .0012 \]
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1. A study of voting chose 663 registered voters at random shortly after an election. Of these, 72% said they had voted in the election. Election records show that only 56% of registered voters voted in the election. Which of the following statements is true about the boldface numbers?
   (a) 72% is a sample; 56% is a population
   (b) 72% and 56% are both statistics
   (c) 72% is a statistic and 56% is a parameter
   (d) 72% is a parameter and 56% is a statistic
   (e) 72% and 56% are both parameters

2. The number of hours a light bulb burns before failing varies from bulb to bulb. The distribution of burnout times is strongly skewed to the right. The central limit theorem says that
   (a) as we look at more and more bulbs, their average burnout time gets ever closer to the mean \( \mu \) for all bulbs of this type.
   (b) the average burnout time of any number of bulbs has a distribution of the same shape (strongly skewed) as the distribution for individual bulbs.
   (c) the average burnout time of any number of bulbs has a distribution that is close to Normal.
   (d) the average burnout time of a large number of bulbs has a distribution of the same shape (strongly skewed) as the distribution for individual bulbs.
   (e) the average burnout time of a large number of bulbs has a distribution that is close to normal.

3. The Gallup Poll has decided to increase the size of its random sample of Canadian voters from about 1500 people to about 4000 people right before an election. The poll is designed to estimate the proportion of Canadian voters who favor a new law banning smoking in public buildings. The effect of this increase is to
   (a) reduce the bias of the estimate.
   (b) increase the bias of the estimate.
   (c) reduce the variability of the estimate.
   (d) increase the variability of the estimate.
   (e) have no effect since the population size is the same.

4. Which of the following statements about the sampling distribution of the sample mean is INCORRECT:
   (a) The standard deviation of the sampling distribution will decrease as the sample size increases.
   (b) The standard deviation of the sampling distribution is a measure of the variability of the sample mean among repeated samples.
   (c) The sample mean is an unbiased estimator of the true (unknown) population mean.
   (d) The sampling distribution shows how the sample mean will vary among repeated samples.
   (e) The sampling distribution shows how the sample was distributed around the sample mean.

5. Suppose we select an SRS of size \( n = 100 \) from a large population having proportion \( p \) of successes. Let \( \hat{p} \) be the proportion of successes in the sample. For which value of \( p \) would it be safe to use Normal approximation to the sampling distribution of \( \hat{p} \)?
   (a) 0.010 \( \times \)
   (b) 0.091 \( \times \)
   (c) 0.850
   (d) 0.975 \( \times \)
   (e) 0.999 \( \times \)
6. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let \( \hat{p} \) denote the proportion in the sample who say they support the increase. Suppose that 40% of all adults in Ohio support the increase. The standard deviation of \( \hat{p} \) is
(a) 0.40
(b) 0.24
(c) 0.0126
(d) 0.00016
(e) 0

7. Suppose we are planning on taking an SRS of size \( n \) from a population. If we double the sample size, then \( \sigma_x \) will be multiplied by
(a) \( \sqrt{2} \)
(b) \( 1/\sqrt{2} \)
(c) 2
(d) 1/2
(e) none of these

8. Which of the following statements is/are true?
\( \sqrt{I} \). The sampling distribution of \( \bar{x} \) has standard deviation \( \sigma/\sqrt{n} \) even if the population is not normally distributed.
\( \sqrt{II} \). The sampling distribution of \( \bar{x} \) is Normal if the population has a Normal distribution.
\( \sqrt{III} \). When \( n \) is large, the sampling distribution of \( \bar{x} \) is approximately Normal even if the population is not normally distributed.

(a) I and II
(b) I and III
(c) II and III
(d) I, II, and III
(e) None of the above gives the correct set of responses.

9. If a statistic used to estimate a parameter is such that the mean of its sampling distribution is different from the true value of the parameter being estimated, the statistic is said to be
(a) Random
(b) Biased
(c) A proportion
(d) Unbiased
(e) None of the above.

10. A machine is designed to fill 16-ounce bottles of shampoo. When the machine is working properly, the mean amount poured into the bottles is 16.05 ounces with a standard deviation of 0.1 ounce. If four bottles are randomly selected each hour and the number of ounces in each bottle is measured, then 95% of the observations should occur in which interval?
(a) 16.05 and 16.15 ounces
(b) -0.30 and +0.30 ounces
(c) 15.95 and 16.15 ounces
(d) 15.90 and 16.20 ounces
(e) None of the above
Part II – Free Response (Question 11-13) – Show your work and explain your results clearly.

11. A Gallup Poll of a random sample of 1089 Canadians (total population of 30,000,000) found that about 80% favored capital punishment. A Gallup Poll of a random sample of 1089 Americans (total population of 300,000,000) also found that 80% favored capital punishment. The accuracy of an estimate describes the closeness of that estimate to the unknown true parameter value. Precision is the degree to which repeated estimates produced using the same sampling method would provide answers very close to each other.

(a) Which poll gives a more accurate estimate of the proportion of that nation’s citizens who favor capital punishment? Justify your answer. 

(b) Which poll gives a more precise estimate of the proportion of that nation’s citizens who favor capital punishment? Justify your answer.

12. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let \( \hat{p} \) denote the proportion in the sample who say they support the increase. Suppose that 40% of all adults in Ohio support the increase.

(a) If \( \hat{p} \) is the proportion of the sample who support the increase, what is the mean and standard deviation of the sampling distribution of \( \hat{p} \)?

\[
\mu_{\hat{p}} = 0.4, \quad \sigma_{\hat{p}} = \sqrt{\frac{0.4 \times 0.6}{1500}} = 0.0126
\]

(b) Using the Binomial distribution, find the probability that \( \hat{p} \) takes a value between 0.37 and 0.43.

\[
P(0.37 < \hat{p} < 0.43) = 0.9916 - 0.0080 = 0.9835
\]

(c) Would it be appropriate to use the Normal approximation to describe the distribution of \( \hat{p} \)? Explain.

\[np = 1500 \times 0.4 = 600, \quad n(1-p) = 1500 \times 0.6 = 900\]

Independence: 15000 < all adults in Ohio

\( \Rightarrow \) Yes

(d) Using the Normal approximation, find the probability that \( \hat{p} \) takes a value between 0.37 and 0.43.

\[
P(0.37 < \hat{p} < 0.43) = 0.982
\]

(e) How large a sample would be needed to guarantee that the standard deviation of \( \hat{p} \) is no more than 0.01? Explain.

\[
s = 0.0126
\]

\[
0.01 \geq \sqrt{\frac{0.4 \times 0.6}{n}} \Rightarrow n \geq 2400
\]
13. A study of college freshmen's study habits found that the time (in hours) that college freshmen study each week is approximately normally distributed with a mean of 7.2 hours and a standard deviation of 5.3 hours.

(a) Calculate the probability that a randomly chosen freshman studies more than 9 hours?

\[ P(x > 9) = 0.367 \]

(b) What is the shape of the sampling distribution of the mean \( \bar{x} \) for samples of 55 randomly selected freshmen? Justify your answer.

The distribution of \( \bar{x} \) is normal since the population is normal.

(c) Would your answer to part (b) change if the population was NOT approximately normal? Explain.

No, because according to CLT, the sample size is sufficiently large enough to make the distribution of \( \bar{x} \) approximately normal.

(d) What are the mean and standard deviation for the average number of hours \( \bar{x} \) spent studying by an SRS of 55 freshmen?

\[ \mu_{\bar{x}} = 7.2 \]  
\[ \sigma_{\bar{x}} = \frac{5.3}{\sqrt{55}} = 0.7147 \]

(e) Find the probability that the average number of hours spent studying by an SRS of 55 students is greater than 9 hours. Show your work.

\[ P(\bar{x} > 9) = 0.0059 \]
Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. A study of voting chose 663 registered voters at random shortly after an election. Of these, 72% said they had voted in the election. Election records show that only 56% of registered voters voted in the election. Which of the following statements is true about the boldface numbers?
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   (d) I, II, and III
   (e) None of the above gives the correct set of responses.

9. If a statistic used to estimate a parameter is such that the mean of its sampling distribution is different from the true value of the parameter being estimated, the statistic is said to be
   (a) Random
   (b) Biased
   (c) A proportion
   (d) Unbiased
   (e) None of the above.

10. A machine is designed to fill 16-ounce bottles of shampoo. When the machine is working properly, the mean amount poured into the bottles is 16.05 ounces with a standard deviation of 0.1 ounce. If four bottles are randomly selected each hour and the number of ounces in each bottle is measured, then 95% of the observations should occur in which interval?
    (a) 16.05 and 16.15 ounces
    (b) -0.30 and +0.30 ounces
    (c) 15.95 and 16.15 ounces
    (d) 15.90 and 16.20 ounces
    (e) None of the above
Part II – Free Response (Question 11-13) – Show your work and explain your results clearly.

11. A Gallup Poll of a random sample of 1089 Canadians (total population of 30,000,000) found that about 80% favored capital punishment. A Gallup Poll of a random sample of 1089 Americans (total population of 300,000,000) also found that 80% favored capital punishment. The accuracy of an estimate describes the closeness of that estimate to the unknown true parameter value. Precision is the degree to which repeated estimates produced using the same sampling method would provide answers very close to each other.
(a) Which poll gives a more accurate estimate of the proportion of that nation’s citizens who favor capital punishment? Justify your answer.

(b) Which poll gives a more precise estimate of the proportion of that nation’s citizens who favor capital punishment? Justify your answer.

12. A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let \( \hat{p} \) denote the proportion in the sample who say they support the increase. Suppose that 40% of all adults in Ohio support the increase.
(a) If \( \hat{p} \) is the proportion of the sample who support the increase, what is the mean and standard deviation of the sampling distribution of \( \hat{p} \)?

(b) Using the Binomial distribution, find the probability that \( \hat{p} \) takes a value between 0.37 and 0.43.

(c) Would it be appropriate to use the Normal approximation to describe the distribution of \( \hat{p} \)? Explain.

(d) Using the Normal approximation, find the probability that \( \hat{p} \) takes a value between 0.37 and 0.43.

(e) How large a sample would be needed to guarantee that the standard deviation of \( \hat{p} \) is no more than 0.01? Explain.
A study of college freshmen's study habits found that the time (in hours) that college freshmen study each week is approximately normally distributed with a mean of 7.2 hours and a standard deviation of 5.3 hours.

(a) Calculate the probability that a randomly chosen freshman studies more than 9 hours?

(b) What is the shape of the sampling distribution of the mean \( \bar{x} \) for samples of 55 randomly selected freshmen? Justify your answer.

(c) Would your answer to part (b) change if the population was NOT approximately normal? Explain.

(d) What are the mean and standard deviation for the average number of hours \( \bar{x} \) spent studying by an SRS of 55 freshmen?

(e) Find the probability that the average number of hours spent studying by an SRS of 55 students is greater than 9 hours. Show your work.
Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. The heights (in inches) of males in the United States are believed to be Normally distributed with mean $\mu$. The average height of a random sample of 25 American adult males is found to be $\bar{x} = 69.72$ inches, and the standard deviation of the 25 heights is found to be $s = 4.15$. The standard error of $\bar{x}$ is

(a) 0.17
(b) 0.69
(c) 0.83
(d) 1.856
(e) 2.04

\[ S_{\bar{x}} = \frac{4.15}{\sqrt{25}} \]

2. You want to estimate the mean SAT score for a population of students with a 90% confidence interval. Assume that the population standard deviation is $\sigma = 100$. If you want the margin of error to be approximately 10, you will need a sample size of

(a) 16
(b) 271
(c) 38
(d) 1476
(e) None of the above

\[ m = 1.64 \left( \frac{100}{\sqrt{n}} \right) \]

3. An analyst, using a random sample of $n = 500$ families, obtained a 90% confidence interval for mean monthly family income for a large population: ($600, $800). If the analyst had used a 99% confidence level instead, the confidence interval would be:

(a) Narrower and would involve a larger risk of being incorrect
(b) Wider and would involve a smaller risk of being incorrect
(c) Narrower and would involve a smaller risk of being incorrect
(d) Wider and would involve a larger risk of being incorrect
(e) Wider but it cannot be determined whether the risk of being incorrect would be larger or smaller

4. The Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. The news report adds: "The poll had a margin of error of plus or minus three percentage points." You can safely conclude that

(a) 95% of all Gallup Poll samples like this one give answers within ±3% of the true population value.
(b) The percent of the population who jog is certain to be between 15% and 21%.
(c) 95% of the population jog between 15% and 21% of the time.
(d) We can be 3% confident that the sample result is true.
(e) If Gallup took many samples, 95% of them would find that exactly 18% of the people in the sample jog.

5. I collect a random sample of size $n$ from a population and from the data collected compute a 95% confidence interval for the mean of the population. Which of the following would produce a new confidence interval with larger width (larger margin of error) based on these same data?

(a) Use a larger confidence level.
(b) Use a smaller confidence level.
(c) Use the same confidence level, but compute the interval $n$ times. Approximately 5% of these intervals will be larger.
(d) Increase the sample size.
(e) Nothing can guarantee absolutely that you will get a larger interval. One can only say the chance of obtaining a larger interval is 0.05.
6. You want to design a study to estimate the proportion of students on your campus who agree with the statement "The student government is an effective organization for expressing the needs of students to the administration." You will use a 95% confidence interval and you would like the margin of error to be 0.05 or less. The minimum sample size required is approximately

- (a) 22
- (b) 1795
- (c) 385
- (d) 271
- (e) None of the above

\[ m_2 \leq 1.96 \sqrt{\frac{.5 \cdot .5}{n}} \]
\[ n \geq 384.16 \]

7. Consider the following graph of the mean yields of barley in 1980, 1984, and 1988 along with 95% confidence intervals.

Which of the following is INCORRECT?

(a) Since the confidence intervals for 1984 and 1980 have considerable overlap, there is little evidence that the sample means differ.

(b) Since the confidence intervals for 1988 and 1980 do not overlap, there is good evidence that their respective population means differ.

(c) The sample mean for 1984 is about 195 g/400 m².

(d) The sample mean for 1988 is less than the sample mean for 1984.

(e) The estimate of the population mean in 1988 is more precise than that for 1980 since the confidence interval for 1988 is narrower than that for 1980.

8. The diameter of ball bearings is known to be Normally distributed with unknown mean and variance. A random sample of size 25 gave a mean of 2.5 cm. The 95% confidence interval had length 4 cm. Then

- (a) the sample variance is 4.86.
- (b) the sample variance is 26.03.
- (c) the population variance is 4.84.
- (d) the population variance is 23.47.
- (e) the sample variance is 23.47.

\[ 2.5 \pm 2 \cdot \frac{s}{\sqrt{25}} \]
\[ 2 = 2.06 \cdot \frac{s}{\sqrt{25}} \]

9. In a poll, (a) some people refused to answer questions, (b) people without telephones could not be in the sample, and (c) some people never answered the phone in several calls. Which of these sources is included in the ±2% margin of error announced for the poll?

- (a) Only source (a).
- (b) Only source (b).
- (c) Only source (c).
- (d) All three sources of error.
- (e) None of these sources of error.

10. Researchers are studying yield of a crop in two locations. The researchers are going to compare two independent 90% confidence intervals for the mean yield in each location. The probability that at least one of the constructed intervals will cover the true mean yield at its location is

- (a) 0.81
- (b) 0.19
- (c) 0.99
- (d) 0.95
- (e) none of these

\[ 1 - (1)^2 \]
Part II - Free Response (Question 11-13) - Show your work and explain your results clearly.

11. Political parties rely heavily upon polling to measure their support in the electorate. At right are the results of a poll conducted in 1996 for four political parties.

(a) Compute the estimated standard error for the level of support of the L party in 1996. Interpret this value.

\[
\hat{p} = \frac{413}{1183} = 0.35 \\
\hat{p} = \sqrt{\left(0.35 \times 0.65 \right) / 1183} = 0.014
\]

(b) Construct and interpret a 95% confidence interval for the level of support for the N party in 1996.

Conditions
SRS - assume true
Independence - \( 1183 \times 10 < \text{all voters} \)
Normality - \( n \hat{p} = (1183)(0.35) = 413 \)
\( n(1-\hat{p}) = (1183)(0.65) = 770 \)

\[
\text{estimate} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

\[
L: 0.35 \pm 1.96 (0.014) \quad (0.322, 0.376)
\]

We are 95% confident that the true proportion of L party supporters is between 0.322 and 0.376.

12. Explain the meaning of the term robust in relationship to inference procedures.

Even in situations where the conditions are not met, \( t \) procedures are robust which means it is still an unbiased estimator to the true population mean.
There are many ways to measure the reading ability of children. Research designed to improve reading performance is dependent on good measures of the outcome. One frequently used test is the DRP, or Degree of Reading Power. A researcher suspects that the mean score \( \mu \) of all third-graders in Henrico County Schools is different from the national mean, which is 32. To test her suspicion, she administers the DRP to an SRS of 44 Henrico County third-grade students. Their scores were:

<table>
<thead>
<tr>
<th>Score</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>26</td>
</tr>
<tr>
<td>47</td>
<td>19</td>
</tr>
<tr>
<td>52</td>
<td>25</td>
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<td>47</td>
<td>35</td>
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<td>14</td>
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<td>25</td>
<td>43</td>
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<td>46</td>
<td>27</td>
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<td>19</td>
<td>26</td>
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<td>35</td>
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<td>46</td>
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<td>26</td>
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<td>41</td>
<td>51</td>
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<td>14</td>
</tr>
<tr>
<td>54</td>
<td>45</td>
</tr>
</tbody>
</table>

The following is a Minitab printout of basic statistics:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>MEAN</th>
<th>MEDIAN</th>
<th>TRMEAN</th>
<th>STDEV</th>
<th>SEMEAN</th>
<th>MIN</th>
<th>MAX</th>
<th>Q1</th>
<th>Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRPscore</td>
<td>44</td>
<td>35.09</td>
<td>35.00</td>
<td>35.25</td>
<td>11.19</td>
<td>1.69</td>
<td>14</td>
<td>54</td>
<td>26.25</td>
<td>44.75</td>
</tr>
</tbody>
</table>

(a) Construct a 90% confidence interval for the mean DRP score in Henrico County Schools.

\[
\bar{x} = 35.09, \quad S_\bar{x} = 1.69, \quad n=44
\]

Conditions:
- SRS - given
- Independence: \( 44 \times 10 > 400 \) 3rd graders in Henrico County
- Normality: large enough
- Sample size so \( n \) by cut

\[
\bar{x} \pm t^* (SE) \quad df=43
\]

\[
35.09 \pm 1.68 (1.69)
\]

\[
(32.25, 37.93)
\]

(b) Use the confidence interval you constructed in (a) to comment on whether you agree with the researcher's claim. Explain your reasoning clearly.

I agree because our confidence interval does not include 32.
Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. The heights (in inches) of males in the United States are believed to be Normally distributed with mean \( \mu \). The average height of a random sample of 25 American adult males is found to be \( \bar{x} = 69.72 \) inches, and the standard deviation of the 25 heights is found to be \( s = 4.15 \). The standard error of \( \bar{x} \) is

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2. You want to estimate the mean SAT score for a population of students with a 90% confidence interval. Assume that the population standard deviation is \( \sigma = 100 \). If you want the margin of error to be approximately 10, you will need a sample size of

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(b) 271  
(c) 38  
(d) 1476  
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3. An analyst, using a random sample of \( n = 500 \) families, obtained a 90% confidence interval for mean monthly family income for a large population: \($600, $800\). If the analyst had used a 99% confidence level instead, the confidence interval would be:

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Part II – Free Response (Question 11-13) – Show your work and explain your results clearly.

11. Political parties rely heavily upon polling to measure their support in the electorate. At right are the results of a poll conducted in 1996 for four political parties.

(a) Compute the estimated standard error for the level of support of the L party in 1996. Interpret this value.

(b) Construct and interpret a 95% confidence interval for the level of support for the N party in 1996.

12. Explain the meaning of the term robust in relationship to inference procedures.
13. There are many ways to measure the reading ability of children. Research designed to improve reading performance is dependent on good measures of the outcome. One frequently used test is the DRP, or Degree of Reading Power. A researcher suspects that the mean score $\mu$ of all third-grade students in Henrico County Schools is different from the national mean, which is 32. To test her suspicion, she administers the DRP to an SRS of 44 Henrico County third-grade students. Their scores were:

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47  19  26  35  34  15  44  40  38  31  46
52  25  35  35  33  29  34  41  49  28  52
47  35  48  22  33  41  51  27  14  54  45

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<td>54</td>
<td>26.25</td>
<td>44.75</td>
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</tbody>
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(a) Construct a 90% confidence interval for the mean DRP score in Henrico County Schools.

(b) Use the confidence interval you constructed in (a) to comment on whether you agree with the researcher's claim. Explain your reasoning clearly.
Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. DDT is an insecticide that accumulates up the food chain. Predator birds can be contaminated with quite high levels of the chemical by eating many lightly contaminated prey. One effect of DDT upon birds is to inhibit the production of the enzyme carbonic anhydrase, which controls calcium metabolism. It is believed that this causes eggshells to be thinner and weaker than normal and makes the eggs more prone to breakage. (This is one of the reasons why the condor in California is near extinction.) An experiment was conducted where 16 sparrow hawks were fed a mixture of 3 ppm dieldrin and 15 ppm DDT (a combination often found in contaminated prey). The first egg laid by each bird was measured, and the mean shell thickness was found to be 0.19 mm. A “normal” eggshell has a mean thickness of 0.2 mm.

The null and alternative hypotheses are

(a) $H_0: \mu = 0.2; H_a: \mu < 0.2$
(b) $H_0: \mu < 0.2; H_a: \mu = 0.2$
(c) $H_0: \bar{x} = 0.2; H_a: \bar{x} < 0.2$
(d) $H_0: \bar{x} = 0.19; H_a: \bar{x} = 0.2$
(e) $H_0: \mu = 0.2; H_a: \mu \neq 0.2$

2. A significance test allows you to reject a hypothesis $H_0$ in favor of an alternative $H_a$ at the 5% level of significance. What can you say about significance at the 1% level?

(a) $H_0$ can be rejected at the 1% level of significance.
(b) There is insufficient evidence to reject $H_0$ at the 1% level of significance.
(c) There is sufficient evidence to accept $H_0$ at the 1% level of significance.
(d) $H_a$ can be rejected at the 1% level of significance.
(e) The answer can’t be determined from the information given.

3. In a test of $H_0: \mu = 100$ against $H_a: \mu \neq 100$, a sample of size 80 produces $t = 0.8$ for the value of the test statistic. The $P$-value of the test is thus equal to

(a) 0.213
(b) 0.426
(c) 0.295
(d) 0.196
(e) 0.165

4. Which of the following is/are correct?

✓ I. The power of a significance test depends on the alternative value of the parameter.
× II. The probability of a Type II error is equal to the significance level of the test.
✓ III. Type I and Type II errors make sense only when a significance level has been chosen in advance.

(a) I and II only
(b) I and III only
(c) II and III only
(d) I, II, and III
(e) None of the above gives the complete set of correct responses.
5. After once again losing a football game to the archrival, a college's alumni association conducted a survey to see if alumni were in favor of firing the coach. An SRS of 100 alumni from the population of all living alumni was taken. 64 of the alumni in the sample were in favor of firing the coach. Suppose you wish to see if a majority of living alumni are in favor of firing the coach. The appropriate test statistic is

(a) \[ z = \frac{(0.64 - 0.5)}{\sqrt{(0.64)(0.36)/100}} \]
(b) \[ z = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.5)/100}} \]
(c) \[ z = \frac{(0.64 - 0.5)}{\sqrt{(0.64)(0.36)/64}} \]
(d) \[ z = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.5)/64}} \]
(e) \[ t = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.64)/100}} \]

6. A noted psychic was tested for ESP. The psychic was presented with 200 cards face down and asked to determine if the card was one of five symbols: a star, cross, circle, square, or three wavy lines. The psychic was correct in 50 cases. Let \( p \) represent the probability that the psychic correctly identifies the symbol on the card in a random trial. Assume the 200 trials can be treated as an SRS from the population of all guesses the psychic would make in his lifetime. Which inference procedure would you use to determine whether the psychic is doing better than just guessing?

(a) one-proportion \( z \) test
(b) one-sample \( t \) test
(c) one-sample \( z \) test
(d) one-proportion \( z \) interval
(e) one-sample \( t \) interval

7. The water diet requires one to drink two cups of water every half hour from when one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weight (in pounds) are:

<table>
<thead>
<tr>
<th>Person</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight before the diet</td>
<td>180</td>
<td>125</td>
<td>240</td>
<td>150</td>
</tr>
<tr>
<td>Weight after the diet</td>
<td>170</td>
<td>130</td>
<td>215</td>
<td>152</td>
</tr>
</tbody>
</table>

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet - weight after six weeks on the diet) is Normally distributed with mean \( \mu \). To determine if the diet leads to weight loss, we test the hypotheses: \( H_0: \mu = 0 \) vs. \( H_a: \mu > 0 \). Based on these data we conclude that

(a) we would not reject \( H_0 \) at significance level 0.10.
(b) we would reject \( H_0 \) at significance level 0.10 but not at level 0.05.
(c) we would reject \( H_0 \) at significance level 0.05 but not at level 0.01.
(d) we would reject \( H_0 \) at significance level 0.01.
(e) none of these

\[ t = 1.027 \]
\[ p = 0.1901 \]

8. Which of the following is not a correct statement about conditions for performing a significance test about an unknown population proportion \( p \)?

(a) The data should come from a randomized experiment or simple random sample.
(b) Individual measurements should be independent of one another.
(c) The population distribution should be approximately Normal, unless the sample size is large.
(d) Both \( np \) and \( n(1 - p) \) should be at least 10.
(e) If you are sampling without replacement from a finite population, then you should sample no more than 10% of the population.
9. To determine the reliability of experts used in interpreting the results of polygraph examinations in criminal investigations, 280 cases were studied. The results were

<table>
<thead>
<tr>
<th>Examiner's Decision</th>
<th>“Innocent”</th>
<th>“Guilty”</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>innocent</td>
<td>131</td>
<td>15</td>
</tr>
<tr>
<td>guilty</td>
<td>9</td>
<td>125</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>140</strong></td>
<td><strong>140</strong></td>
</tr>
</tbody>
</table>

If the hypotheses were $H_0$: suspect is innocent vs. $H_a$: suspect is guilty, then we could estimate the probability of making a Type II error as

(a) $15/280$
(b) $9/280$
(c) $15/140$
(d) $9/140$
(e) $15/146$

10. A 95% confidence interval for $\mu$ is calculated to be $(1.7, 3.5)$. It is now decided to test the hypothesis $H_0: \mu = 0$ versus $H_a: \mu \neq 0$ at the $\alpha = 0.05$ level, using the same data as used to construct the confidence interval.

(a) We cannot test the hypothesis without the original data.
(b) We cannot test the hypothesis at the $\alpha = 0.05$ level since the $\alpha = 0.05$ test is connected to the 97.5% confidence interval.
(c) We can make the connection between hypothesis tests and confidence intervals only if the sample sizes are large.
(d) We would reject $H_0$ at level $\alpha = 0.05$.
(e) We would accept $H_0$ at level $\alpha = 0.05$.

Part II – Free Response (Question 11-13) – Show your work and explain your results clearly.

11. Eleven percent of the products produced by an industrial process over the past several months fail to conform to the specifications. The company modifies the process in an attempt to reduce the rate of nonconformities. In a trial run, the modified process produces 16 nonconforming items out of a total of 300 produced.

Do these results demonstrate that the modification is effective?

\[
H_0: p = 0.11 \\
H_a: p < 0.11 \\
\alpha = 0.05 \\
\hat{p} = \frac{16}{300} = 0.053
\]

Since our p-value is less than our alpha level, we reject the null hypothesis. We have reason to believe the modification is effective in reducing the rate of nonconformities.
12. The nicotine content in cigarettes of a certain brand is Normally distributed with mean $\mu = 1.5$ (in milligrams) and standard deviation $\sigma = 0.2$ mg. The brand advertises that the mean nicotine content of their cigarettes is 1.5 mg, but you are suspicious and plan to investigate the advertised claim by testing the hypotheses

$$H_0 : \mu = 1.5 \text{ versus } H_a : \mu > 1.5$$

at the $\alpha = 0.05$ significance level. You will do so by measuring the nicotine content of 30 randomly selected cigarettes of this brand and computing the mean nicotine content $\bar{x}$ of your measurements.

(a) Describe a Type I and a Type II error in this setting, and give the consequences of each.

Type I: We say the nicotine content is higher than 1.5 mg when in fact it is not. The company has negative publicity and angry customers.

Type II: We say the nicotine content is 1.5 mg but it is in fact higher. The company is open to lawsuit for poisoning its customers.

(b) Find the power of the test if $\mu = 1.6$ mg.

$$\sigma = \frac{0.2}{\sqrt{30}}$$

$\bar{x}$

$\mu$

$H_0$

$H_a$

power = $P(\bar{x} > 1.56) = 0.863$

(c) Describe two different types of changes you could make to increase the power of the test in the previous question.

Increase alpha level
Increase sample size
Part I - Multiple Choice (Questions 1-10) – Circle the letter of the answer of your choice.

1. DDT is an insecticide that accumulates up the food chain. Predator birds can be contaminated with quite high levels of the chemical by eating many lightly contaminated prey. One effect of DDT upon birds is to inhibit the production of the enzyme carbonic anhydrase, which controls calcium metabolism. It is believed that this causes eggshells to be thinner and weaker than normal and makes the eggs more prone to breakage. (This is one of the reasons why the condor in California is near extinction.) An experiment was conducted where 16 sparrow hawks were fed a mixture of 3 ppm dieldrin and 15 ppm DDT (a combination often found in contaminated prey). The first egg laid by each bird was measured, and the mean shell thickness was found to be 0.19 mm. A "normal" eggshell has a mean thickness of 0.2 mm.

The null and alternative hypotheses are
(a) $H_0: \mu = 0.2; H_a: \mu < 0.2$
(b) $H_0: \mu < 0.2; H_a: \mu = 0.2$
(c) $H_0: \bar{x} = 0.2; H_a: \bar{x} < 0.2$
(d) $H_0: \bar{x} = 0.19; H_a: \bar{x} = 0.2$
(e) $H_0: \mu = 0.2; H_a: \mu \neq 0.2$

2. A significance test allows you to reject a hypothesis $H_0$ in favor of an alternative $H_a$ at the 5% level of significance. What can you say about significance at the 1% level of significance?
(a) $H_0$ can be rejected at the 1% level of significance.
(b) There is insufficient evidence to reject $H_0$ at the 1% level of significance.
(c) There is sufficient evidence to accept $H_0$ at the 1% level of significance.
(d) $H_a$ can be rejected at the 1% level of significance.
(e) The answer can't be determined from the information given.

3. In a test of $H_0: \mu = 100$ against $H_a: \mu \neq 100$, a sample of size 80 produces $t = 0.8$ for the value of the test statistic. The $P$-value of the test is thus equal to
(a) 0.213
(b) 0.426
(c) 0.295
(d) 0.196
(e) 0.165

4. Which of the following is/are correct?
   I. The power of a significance test depends on the alternative value of the parameter.
   II. The probability of a Type II error is equal to the significance level of the test.
   III. Type I and Type II errors make sense only when a significance level has been chosen in advance.

   (a) I and II only
   (b) I and III only
   (c) II and III only
   (d) I, II, and III
   (e) None of the above gives the complete set of correct responses.
5. After once again losing a football game to the archrival, a college’s alumni association conducted a survey to see if alumni were in favor of firing the coach. An SRS of 100 alumni from the population of all living alumni was taken. 64 of the alumni in the sample were in favor of firing the coach. Suppose you wish to see if a majority of living alumni are in favor of firing the coach. The appropriate test statistic is
(a) \( z = \frac{(0.64 - 0.5)}{\sqrt{(0.64)(0.36)/100}} \)
(b) \( z = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.5)/100}} \)
(c) \( z = \frac{(0.64 - 0.5)}{\sqrt{(0.64)(0.36)/64}} \)
(d) \( z = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.5)/64}} \)
(e) \( t = \frac{(0.64 - 0.5)}{\sqrt{(0.5)(0.64)/100}} \)

6. A noted psychic was tested for ESP. The psychic was presented with 200 cards face down and asked to determine if the card was one of five symbols: a star, cross, circle, square, or three wavy lines. The psychic was correct in 50 cases. Let \( p \) represent the probability that the psychic correctly identifies the symbol on the card in a random trial. Assume the 200 trials can be treated as an SRS from the population of all guesses the psychic would make in his lifetime. Which inference procedure would you use to determine whether the psychic is doing better than just guessing?
(a) one-proportion \( z \) test
(b) one-sample \( t \) test
(c) one-sample \( z \) test
(d) one-proportion \( z \) interval
(e) one-sample \( t \) interval

7. The water diet requires one to drink two cups of water every half hour from when one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weight (in pounds) are:

<table>
<thead>
<tr>
<th>Person</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight before the diet</td>
<td>180</td>
<td>125</td>
<td>240</td>
<td>150</td>
</tr>
<tr>
<td>Weight after the diet</td>
<td>170</td>
<td>130</td>
<td>215</td>
<td>152</td>
</tr>
</tbody>
</table>

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet - weight after six weeks on the diet) is Normally distributed with mean \( \mu \). To determine if the diet leads to weight loss, we test the hypotheses: \( H_0: \mu = 0 \) vs. \( H_1: \mu > 0 \). Based on these data we conclude that
(a) we would not reject \( H_0 \) at significance level 0.10.
(b) we would reject \( H_0 \) at significance level 0.10 but not at level 0.05.
(c) we would reject \( H_0 \) at significance level 0.05 but not at level 0.01.
(d) we would reject \( H_0 \) at significance level 0.01.
(e) none of these

8. Which of the following is not a correct statement about conditions for performing a significance test about an unknown population proportion \( p \)?
(a) The data should come from a randomized experiment or simple random sample.
(b) Individual measurements should be independent of one another.
(c) The population distribution should be approximately Normal, unless the sample size is large.
(d) Both \( np \) and \( n(1 - p) \) should be at least 10.
(e) If you are sampling without replacement from a finite population, then you should sample no more than 10% of the population.